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Contagious Defaults in Interbank Networks

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Abstract

This paper investigates systemic risk and contagion processes in an interbank network using network science methods. The interbank network is studied to understand the contagion process within a network considering differences in the network structure and the characteristics of components. Simulations support the claim that heterogeneous networks are more resilient to contagious shocks, while these shocks are more problematic in homogeneous networks. This paper also shows that more interconnections among banks could accelerate or block the contagion process, depending on the structure of the network and the seniority of debts in the interbank network.

1. Introduction

In recent decades, globalization and free markets have increased interdependencies among financial institutions worldwide, creating a more complex financial structure. After the global financial crisis of 2007-2008, which played a significant role in the failure of different financial institutions, many concerns arose in academia, industry, and regulatory bodies about the stability of complex financial systems and possible future systemic failures. Since then, a sizeable body of literature has concentrated on measuring systemic risk and preventing systematic failure before it passes critical thresholds.

Systemic risk is the keyword in studying financial networks. It generally refers to a cascade of failures, or in other words, the financial domino effect caused by a shock in the system. Systemic risk could result from an aggregate negative shock through the system affecting individual entities. It means that it is not only about single risky events but also a series of adverse events correlated with each other in a system. The interbank market plays an essential role within financial systems as it is a system of banks connected by credit links. Financial distresses such as insolvency and illiquidity when an institution cannot fulfill its financial obligations may result in a contagion process affecting other institutions within the network. For instance, Mitchener and Richardson (2019) document the role of the interbank network in the amplification of financial shocks during the Great Depression.

Haldane and May (2011) show how systemic risk is affected by the source of initial shocks and the financial network structure. They show that the degree of vulnerability of an individual financial institution and the network structure are decisive factors in studying systemic risk in banking ecosystems. The latter is

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closely related to the degree of interconnectedness within interbank networks. On the one hand, a higher interconnectedness of the interbank network can mitigate the shock by diffusing it throughout the system among several network members. On the other hand, a higher interconnected network means a larger part of the network is exposed to the shock if the interconnectedness passes a critical threshold, resulting in more defaults. An externality arising from a high degree of interconnectedness among financial institutions, particularly banks, is *too connected to fail* (TCTF) problem alongside the *too big to fail* (TBTF) problem. The TCTF paradigm refers to the failure of institutions with a high degree of centrality in a financial network. Leventides et al. (2019) study the resilience of financial networks with different structures and a high degree of centrality to exogenous shocks. They show that networks with a higher level of concentration around the largest nodes are less stable during financial distress.

In their seminal paper, Allen and Gale (2000) show how small shocks can be amplified and significantly affect interbank markets with incomplete network structures. Similarly, Georg (2013), Montagna and Lux (2017) confirm that interbank network structure has a crucial impact on financial stability in times of distress. However, there are studies arguing that the network topology does not play a role in the propagation of risk and resilience of the interbank network (e.g., Krause and Giansante 2012; Birch and Aste 2014).

This paper investigates the contagion process within a weighted directed interbank network in line with the abovementioned studies. Financial institutions are interconnected through the interbank lending market within a network, which can take two different topologies: scale-free resembling a real-world network and a dense network with a higher degree of interconnectedness. Employing network measures and epidemic modeling, simulations in this paper analyze the contagion process throughout the network in the case of the largest institution's default.

The key finding of this work is twofold. First, regardless of the network structure, a network consisting of banks with heterogeneous characteristics is more resilient to contagion and systemic defaults when the seniority level of the debt is the same for all institutions in the network. Second, the results show that a heterogeneous network with components with different financial characteristics is resilient to contagion processes when the banks are extensively interconnected. In contrast, a dense network of homogeneous banks with the same financial metrics but a higher degree of interconnectedness is more vulnerable to systemic shocks and failures.

The contribution of this work is the presented model allowing for two layers of linkages (senior debt and subordinated debt) between financial institutions inside the network. The most related study to this paper is Bargigli et al. (2015), which proposes a multiplex network model of the Italian banking system in which layers differ due to different maturity times. This paper, however, separates network linkages based on debt seniority.

The paper proceeds as follows. The following section reviews theoretical and empirical contributions to the literature, emphasizing the network-based models. Section 3 presents the model. The results and findings are discussed in the subsequent section. Section 5 draws some conclusions and discusses future avenues for research.

2. Relevant Literature

There is a growing literature measuring systemic risk and contagion in the financial network, particularly interbank networks. The literature follows different strands. Part of the literature mainly concentrates on the theoretical foundations of systemic risk within financial markets. A significant number of these studies show that while the interbank network can absorb financial shocks in the case of proper liquidity management mechanism, it can also amplify the shock within the economy when the network is incomplete (Allen and Gale 2000; Allen et al. 2012; Eisenberg and Noe 2001; Gai et al. 2011; Thurner et al. 2003; Rochet and Tirole 1996;). Moreover, studying credit lines in interbank networks, Orhun (2017) shows that given the optimal liquidity network of Allen and Gale (2000), the interbank network is robust even when the banks face an aggregate liquidity shock.

This paper relates to the strand of the literature that applies network analysis to financial networks. This area of research is mainly concerned with the effect of network topology on the contagion process. Studying contagion in a simulated financial network, Nier et al. (2007) show that at low levels of connectivity, an increase in connectivity would increase the chance of systemic failure. In contrast, high connectivity levels improve the banking system's ability to absorb shocks to prevent systemic failure. On a related note, Klinger and Teply (2016) investigate the relationship between the financial system` and sovereign debt crises by constructing an agent-based network model of an artificial financial system. The authors show that in the short term, all support measures improve the systemic stability of the interbank network, and in the longer run, some settings mitigate the systemic risk.

Several studies also propose various network measurements and indicators for connectedness and systemic risk in interbank networks. Combining variance decomposition of vector autoregressions (VARs) and network topology theory, Diebold and Yılmaz (2014) propose different connectedness measures in interbank networks, particularly among major U.S. financial institutions. The authors show that these measures are intimately related to key measures of connectedness in the network literature and can measure systemic risk. Following the same approach, Baruník and Křehlík (2018) introduce a framework based on the spectral representation of variance decomposition and connectedness measures. The authors monitor shocks at different frequencies to assess their effects on system-wide connectedness or systemic risk. The results show that shocks to one asset in the system will have a long-term effect when connectedness is created at lower frequencies, while in the case of connectedness at higher frequencies, shocks will have a short-term impact.

Moreover, Montagna and Lux (2017) study systemic risk in simulated scale-free interbank networks. The authors show how the net worth ratio on total assets and interbank assets on total assets affect the spread of an idiosyncratic shock. The results indicate that a shell structure in the diffusion of losses in the network, i.e., creditor banks of the defaulted entity, mostly fails before the others, and it is possible to classify the defaults of the different shells in the cascade events. They also find that random networks or networks based on a maximum entropy principle lead to fewer contagious defaults than scale-free networks. This finding is in contrast with the results in previous studies. For instance, comparing different interbank structures, Georg (2013) argues that random networks are more vulnerable to systemic risk than interbank networks with few large and many smaller banks.

This paper is also closely related to the strand of literature using network structures and balance sheets to study systemic risk and contagion processes in different countries' financial/interbank networks. Boss et al. (2004) study the impact of network structure on the stability of the Austrian interbank market. They suggest two significant general results: first, the interbank network is a small world with a very low degree of separation between any two nodes in the system; second, a more realistic class of scale-free networks must be used for future modeling interbank relations. Similarly, Cajueiro and Tabak (2008) and Iori et al. (2008) investigate the Brazilian and Italian interbank networks, respectively. They also find a scale-free structure for these markets. There are few banks with a high level of interconnection and a more significant number of small banks with few connections. They also investigate the characteristics of the nodes to understand the role of the different types of banks in the interbank network. They show that smaller private domestic retail banks play a crucial role as creditors in the stability of the interbank network.

Using a worldwide dataset, Cihak et al. (2012) show that an increase in the degree of interconnectedness in a country's banking system that is less connected to the international banking network is associated with a lower probability of a systemic crisis. However, once the degree of interconnectedness passes a certain threshold, further increases in interconnectedness will negatively affect network stability. In a similar study focusing on the interbank markets in Central and Eastern Europe, Fiala and Havranek (2017) show that the degree of interconnectedness in the local interbank market is the main amplifier of the contagion compared to the linkages to the foreign markets. Moreover, Tonzer (2015) studies the cross-border contagion in interbank networks and finds that in stable times, a higher level of interconnectedness in the banking system can help network resilience in the case of positive spillover effects. In contrast, studying the European interbank network, Gabrieli and Salakhova (2019) show that a denser network with a shorter average path is less resilient to contagion, while a higher level of clustering increases network stability. There are other studies showing the importance of the network structure in different national and international interbank networks (see, e.g., Aldasoro and Alves 2018; Chen et al. 2020; Erkol et al. 2016; Kanno 2015; Langfield et al. 2014; Liu et al. 2020; Martinez-Jaramillo et al. 2014; Müller 2006; Philippas et al. 2015; Silva et al. 2016). One may consult Upper (2011) and Silva et al. (2017) for a comprehensive summary of contagion simulations in different interbank networks.

Finally, another stream of research related to this work uses epidemic modeling for studying systemic risk in financial networks. Blume et al. (2011) study the interbank network as an epidemic in this respect. They show that issues such as the trade-offs between clustered and anonymous market structures expose a fundamental dynamic in which minimal amounts of overlinking in financial networks with contagious risk can have substantial consequences for possible future systemic failures. Consistent with the TCTF problem, Brandi et al. (2018) further show that the riskiness of a bank is better captured by its network centrality. Among others, Bucci et al. (2019), Dehshalie et al. (2021), Hurd et al. (2017), and Toivanen (2013) employ epidemiological methods to study systemic risk and contagion within various financial and interbank networks.

3. Model

The model presented in this paper is a weighted directed interbank network consisting of nodes representing banks. Banks are interconnected by their simplified balance sheets. To have an empirically reliable model, the characteristics of the network resemble the real-world interbank network. Hence, in the formation of the network, the size of assets and liabilities of the banks are similar to those of the most prominent Italian banks on two criteria: first, having total assets of at least 20 billion Euros; second, not being a subsidiary or branch of a foreign-based institution. Based on these conditions, the interbank network consists of 10 banks. It is also assumed that larger financial institutions tend to borrow (lend) from (to) institutions of the same size. Similar to the Italian interbank network, two systemically important nodes have the most significant amount of capital and the most considerable exposure to other banks in the network (i.e., Intesa Sanpaolo and UniCredit). There are other eight banks ranked by their total assets.

Table 1

assets A_i	*liabilities L_i*
liquid assets α_i	senior debt b_j^s
illiquid assets β_i senior loans l_i	junior debt b_i^j
subordinated loans l_i^j	net worth W_i

Notes: Balance sheet of bank *i* regarding its interbank connections. Assets can be written as $A_i = \alpha_i + \beta_i + l_i^s + l_i^j$ and liability is $L_i = b_i^s + b_i^j$. Besides, $W_i = A_i - L_i$ is the net worth of bank *i*.

As shown in Table 1, it is assumed that each bank has two levels of liability on its balance sheet: senior debt $b^s$and junior (subordinated) debt b^j. These two levels of liabilities make an interbank network with two layers of connections. However, since interbank exposures are assumed to be senior, the main analysis focuses only on the senior layer of this network. On the assets side, there are four elements: liquid assets α, illiquid assets β, senior loans l^s, and subordinated loans l^j. The total assets and total liabilities of bank i are A_i and L_i, respectively; and the value of all assets after paying off the liabilities is the net worth of the bank denoted by W_i. Assuming two banks, a and b, there is a link $a \to b$ implying a loan l_{ab} made by bank a to bank b, and the opposite exposure is shown by l_{ba}. In general, the sum of all weights flowing into bank i is $b_i = \sum_{j \in banks} l_{ji}$, which is the total liabilities of bank i in the interbank network. On the other hand, the sum of all weights flowing out of bank i, $l_i = \sum_{j \in banks} l_{ij}$, is the total interbank claims of bank i, where for both b_i and l_i, $j \neq i$.

Each bank in the network uses its liabilities L_i to invest at interest rate R_i that can be equal to or larger than one concerning the investment conditions. The profit made in this transaction can be shown as $\rho_i = (R_i - 1) L_i$. Following Smerlak et al. (2015), there are three different solvency conditions for a bank to pay off its liabilities:

1. Solvent: A bank is solvent if

$$\alpha_i + \rho_i + \sum_{j \neq i} x_{ij} \geq r b_i^s + r b_i^j, \tag{1}$$

where $x_{ji} = r\left(l_{ij}^s + l_{ij}^j\right)$. The interbank borrowing rate is equal to the risk-free rate denoted by r. l_{ij}^s and l_{ij}^j are the loan made by bank i to bank j in senior and subordinated forms, respectively. The equation means that bank i pays off its liabilities in full.

2. Partial solvent: Bank i is partially solvent and pays off a fraction of its liabilities if

$$0 < \alpha_i + \rho_i + \sum_{j \neq i} x_{ij} - r b_i^s < r b_i^j,$$

Therefore, for the amount repaid by bank i to bank j, we have

$$x_{ij} = \frac{l_{ij}}{b_i^j}(\alpha_i + \rho_i - b_i^s + \sum_{j \neq i} x_{ij}).$$

3. Insolvent: Bank i cannot pay off any part of its liabilities, which may lead to default if

$$\alpha_i + \rho_i + \sum_{j \neq i} x_{ij} \leq r b_i^j,$$

Hence, $x_{ij} = 0$ for each $j \neq i$. After all payments, the updated net worth of bank i is

$$W_i = \alpha_i + \beta_i + \rho_i - b_i^s + \sum_{j \neq i}\left(x_{ij} - x_{ji}\right),$$

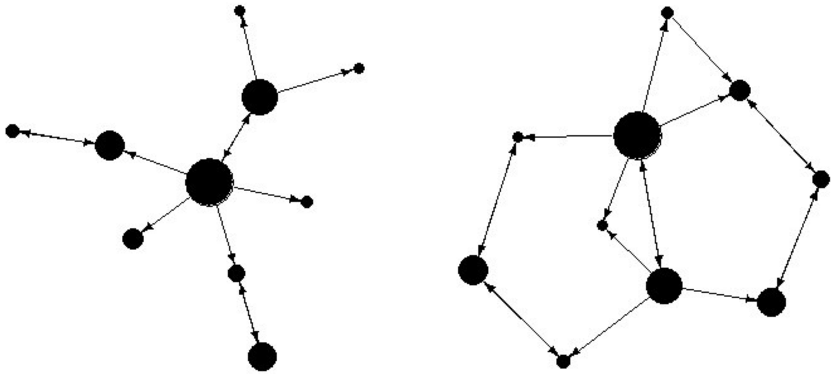
where x_{ij} and x_{ji} imply the repayment of junior loans. Bank i is in the *safe* mode if $W_i > 0$ or *failed* if $W_i \leq 0$.

Table 2 Network Measures for the Benchmark Networks Before Shocks and Individual Defaults

Measures	*Scale-free network*	*Dense network*
Density	0.13	0.20
Average Degree	2.40	3.60
ASPL	1.54	1.48
Reciprocity	0.50	0.56
Assortativity	-0.07	-0.35

Figure 1 Scale-Free and Dense Networks

(a) Scale-free network *(b) Dense network*



Notes: Two directed networks of 10 banks, which are formed with two different methods. The left one is a scale-free network, while the right one is a denser network with more connections between the nodes. The size of a node is related to its net worth (W_i).

The interbank network takes two different formations. The first resembles a scale-free network with few banks with a high level of interconnection and a more significant number of small banks with few connections. Since the two largest banks in the Italian interbank network form two clusters, scale-free is a reasonable topology for the interbank market modeled in this study. Moreover, several studies show that real-world interbank networks usually exhibit characteristics similar to scale-free topology (Boss et al. 2004; Degryse and Nguyen 2007; Georg 2013). The formation of a scale-free network is based on a *power-law* degree distribution:

$$p_k = k^{-\gamma}.$$

The parameter γ can take different numbers depending on the research context. De Masi et al. (2006) show that this parameter is between 2.15 and 2.7 in the Italian interbank network. Moreover, González-Avella et al. (2016) document that the reported value for γ in the economics and finance literature is typically between 2 and 3. Following the suggested values in the literature, particularly those for the Italian interbank market, γ equals 2.5 in forming the interbank network presented in this study.

The second network, *dense* network, is a denser network on a relative basis. This topology exhibits a higher degree of interconnectedness between banks, which means a denser and more connected network than the scale-free network. Unlike scale-free or random networks, the dense network is arbitrarily formed. Studying this type of network aims to assess the role of a higher degree of linkage in an interbank network compared to interbank networks that are more frequently observed in the real world, i.e., scale-free networks. One can check that there are repeating patterns, both triadic and 5-element motifs, in the second network, while there is no repeating pattern in the first network. Figure 1 depicts both networks, and Table 2 summarizes their properties. Moreover, despite differences, both networks' characteristics resemble the actual interbank network characteristics. For example, both networks have a low level of negative assortativity., which is consistent with previous studies on real financial networks (Soramäki et al., 2007).

3.1 Solvent-Insolvent (SI) Epidemic Model

In addition to the primary balance sheet-based model, this study employs a basic epidemic model to investigate interbank contagion. Epidemic models capture the dynamics of the propagation of disease within a community. This class of models addresses questions regarding the spread of an event (e.g., disease or default) in various networks, such as social networks, the population in a country, and interbank networks. Of the most well-known epidemic methods, susceptible-infected (SI), susceptible-infected-recovered (SIR), and susceptible-infected-susceptible (SIS) are the most used in different fields of research. For instance, in a recent study, Fukui and Furukawa (2020) document the properties of coronavirus outbreaks by using a stochastic SIR model, which can be insightful for studies focusing on the dynamics of interbank contagion.

Kermack and McKendrick (1927) introduce a simple epidemic model to study disease spreading among population networks. Although investigating disease spreading or event spreading in a network is a naive method, it is still commonly used in the scientific literature. In this two-stage method, there are two kinds of nodes in the network: susceptible and infected. Once one becomes infected, susceptible nodes become infected, and the disease will spread throughout the entire network. The model can be written as:

$$\frac{dX}{dt} = \frac{\beta SX}{n},$$

$$\frac{dX}{dt} = -\frac{\beta SX}{n},$$

where S is susceptible, and X is infected. Besides, β is the infection rate and can be translated to the probability of contagion in the network. Parameters are defined such that

$$s = \frac{S}{n}, \;\; x = \frac{X}{n}, \;\; s = 1 - x.$$

Then we have a reduced model:

$$\frac{dX}{dt} = \beta(1 - x)x. \tag{2}$$

The solution to this logistic growth equation leads to

$$x(t) = \frac{x_0 \, e^{\beta t}}{1 - x0 + x_0 \, e^{\beta t}}$$

In the context of financial markets, one can interpret the concept in financial jargon as *solvent-insolvent*. At any point in time, an individual bank is either solvent or insolvent (default). Suppose that a bank is susceptible to a specific default probability. Bilateral exposures that connect banks are transmission channels to diffuse the shock among different banks in the network. To catch the contagion process, one of the connected banks must initially become insolvent. Since this insolvent bank transmits the so-called disease at the rate β, the probability that another bank becomes insolvent can be obtained using Equation (2) with still solvent banks.

Translating the SI framework into a solvent-insolvent framework for the interbank network presented in this study is trivial. First, having the same characteristics, banks form a homogeneous interbank network. In other words, it is assumed that banks exhibit identical balance sheets. Second, the default probability of the banks can be interpreted as an infection rate. Although this simple solvent-insolvent framework can be a helpful tool for investigating contagion in an interbank network, it has some drawbacks facing real-world simulations. For example, an insolvent financial institution does not suddenly declare bankruptcy, and it could use the extra funding to repay its obligations.

4. Results

Removing a bank initiates the shock in the network. This shock applies to the most systemically important component in the network: the bank with the highest number of connections to other banks. This shock implies the worst-case scenario in the system. This scenario implies that Equation (1) does not hold for the largest bank of the network, resulting in the bank's default. The default of the largest bank will further cause a cascade of failures, and consequently, it could lead the network to a systemic default.

For the benchmark model, there is a homogeneous ratio of the junior debt to the senior debt in the network, which is $1/9$. The interbank interest rate and interest rate on subordinated debt are equal to the risk-free interest rate, 0.01, while the net interest rate on investment is 0.02. There is also a random number for the percentage of the banks' liquid assets, which is normally distributed as $\alpha_i \sim N(0.08, 0.02)$ of L_i. Finally, the ratio of interbank borrowings to the total senior liabilities $(\frac{b^s_{interbank}}{b^s})$ for each bank is fixed at 0.5.

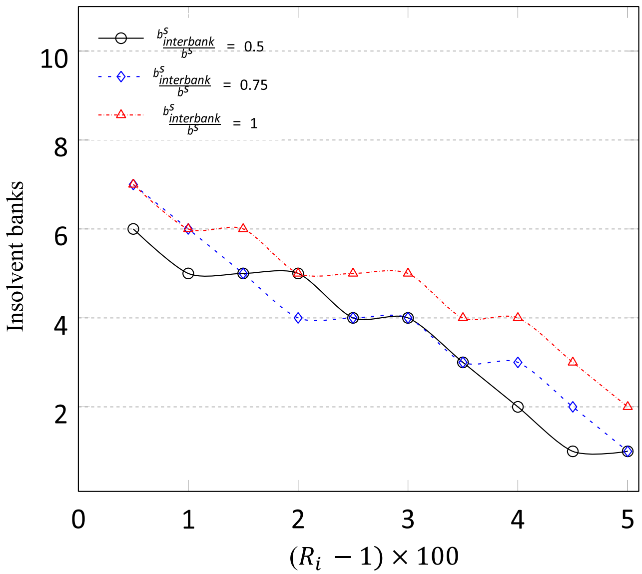
The result of the benchmark simulation for the scale-free network is shown in Table 3. This type of network exhibits a fragile structure due to the shock initiated by the largest component's default—the initial shock results in the default of 4 other banks in the network. The network density further falls dramatically from 0.13 to 0, which means the remaining solvent banks are not connected as the density is virtually zero. In addition, the simulation is repeated for different values of R_i and $(\frac{b^s_{interbank}}{b^s})$. The results depicted in Figure 2 show that the higher the ratio of interbank borrowing to the senior debt, the higher number of insolvent banks. In contrast, the number of failures decreases when the interest rate R_i and the profit made from investment ρ_i increase. For example, in the case of an interest rate equal to 0.05 and $\frac{b^s_{interbank}}{b^s} = 0.5$, the largest bank's default will not make other banks in the network insolvent.

Table 3 The Result of the Benchmark Simulation for the Scale-Free Network

Scale-free Network $R_i - 1 = 0.02$	$\frac{b^S_{interbank}}{b^s} = 0.5$	*Initial*	*Shock*	*After shock*
Insolvent nodes		0	1	5
Density		0.13	0.08	0
Average Degree		2.40	1.30	0
ASPL		1.54	1	0
Reciprocity		0.50	0.69	0
Assortativity		-0.07	None	None

Notes: Changes in network measures before, during, and after shock with respect to $R_i = 0.02$ and $\frac{b^s_{interbank}}{b^s} = 0.5$.

Figure 2 The Number of Insolvent Banks if the Ratio of Interbank Debt to Total Senior Liabilities and Interest Rate Change in the Benchmark Scale-Free Network

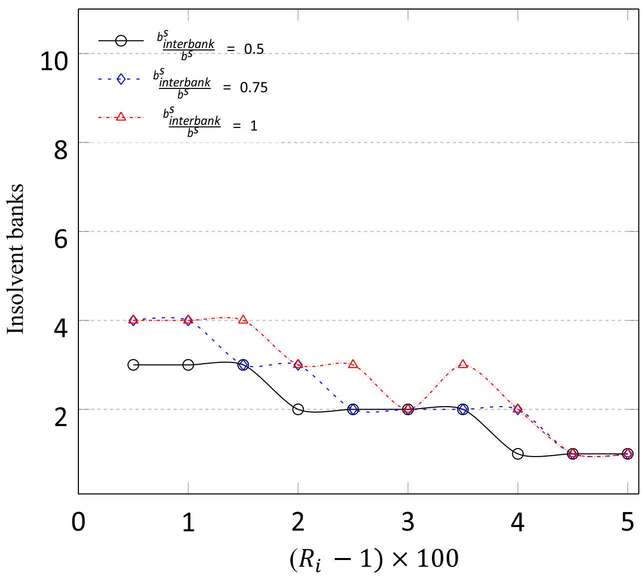


On the other hand, the dense network is more stable than the scale-free network. The benchmark results in Table 4 imply no severe systemic contagion in the network as only one other bank becomes insolvent because of the first bank's insolvency. It means that in the case of the default of important banks in the dense interbank network with an adequate number of interconnections, the probability of contagion and systemic default is low. In other words, the simulation confirmed the idea that more connections in a network reduce the risk of contagion and systemic default. These results are in line with a strand of the network documenting the resilience of dense interbank networks with a high degree of interconnectedness (Georg, 2013). Similar to the scale-free network, changing the ratio of interbank borrowing to the total senior liabilities and the profit made from investments affect the stability of the network. Figure 3 shows that the number of failures in the dense network decreases in the presence of higher values of R_i and lower values of $\frac{b^S_{interbank}}{b^S}$.

Table 4 Changes in Network Measures Before, During, and After Shock with Respect to $\mathbf{R_i = 0.02}$ and $\mathbf{\frac{b^s_{interbank}}{b^s} = 0.5}$

Dense Network $R_i - 1 = 0.02$	$\frac{b^S_{interbank}}{b^S} = 0.5$	*Initial*	*Shock*	*After shock*
Insolvent nodes		0	1	2
Density		0.20	0.16	0.16
Average Degree		3.60	2.66	2.66
ASPL		1.48	1.59	1.59
Reciprocity		0.56	0.66	0.66
Assortativity		-0.35	-0.37	-0.37

Figure 3 The Number of Insolvent Banks if the Ratio of Interbank Debt to Total Senior Liabilities and Interest Rate Change in the Benchmark Dense Network



Furthermore, sensitivity analyses are used to examine the preliminary results. To this end, first, both networks are reconstructed 500 times with identical balance sheet characteristics to the benchmark model but with different arbitrary interbank claims. The network properties differ in each simulation. The average value and standard deviation for each network measure are reported in Table 5. On average, one can check that all assumptions about the two networks' differences still hold. Second, every constructed network encounters a shock by removing the largest component with the highest number of connections, and network resilience is examined by using various values of R_i and $\frac{b^s_{interbank}}{b^s}$.

Table 5 Average Network Measures Before Applying Shocks and Individual Defaults, Drawn from 500 Simulations

Measures	*Scale-free network*	*Dense network*
Density	0.18	0.28
	(0.06)	(0.10)
Average Degree	2.99	4.50
	(0.57)	(0.87)
ASPL	1.99	1.70
	(0.29)	(0.28)
Reciprocity	0.59	0.65
	(0.23)	(0.33)
Assortativity	-0.051	-0.35
	(0.10)	(0.21)

Notes: Standard deviations are in parentheses.

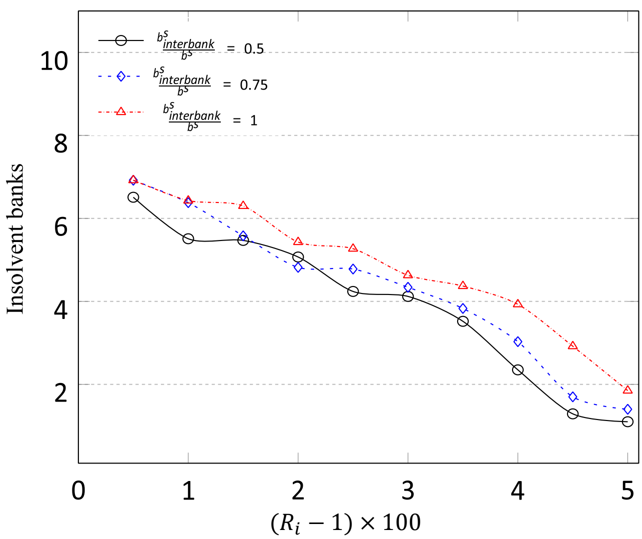
In line with the benchmark model, the results indicate that the ratio of interbank borrowing to senior debt is positively associated with the number of insolvent banks in the network due to the initial default shock. On the contrary, the interest rate R_i negatively affects the number of insolvent banks in the network. In other words, the higher external profit available for the bank helps the bank cover its losses due to systemic contagion in the network. Tables 6 and 7 show the results of the default of the largest bank in the scale-free network and dense network, respectively. The interest rate and the ratio of interbank borrowing to total senior liabilities, not surprisingly, have opposite impacts on the number of failures in the network for both networks. However, the results confirm that the scale-free network is more fragile to default shock than the dense network. Ranging from 1.05 to 2.09, the ratio of insolvent banks in the scale-free network to the dense network indicates that the dense network is more stable in any comparative scenario. The results are similar to the common findings in the literature (e.g., Lenzu and Tedeschi 2012; Montagna and Lux 2017). Moreover, comparing Figures 4 and 5, one can confirm that in the case of dense network, the rate of change in the number of insolvent banks due to changes in R_i is flatter, which can be interpreted as a more stable network structure in comparison with the scale-free network.

Table 6 Average Number of Insolvent Banks Due to Changes in the Ratio of Interbank Debt to Total Senior Liabilities and Interest Rate

$(R_i - 1) \times 100$	$\frac{b^s_{interbank}}{b^s} = 0.50$	$\frac{b^s_{interbank}}{b^s} = 0.75$	$\frac{b^s_{interbank}}{b^s} = 1$
	# of insolvent banks	*# of insolvent banks*	*# of insolvent banks*
0.5	6.51 (0.80)	6.92 (0.88)	6.92 (0.84)
1	5.51 (0.81)	6.38 (1.21)	6.43 (0.59)
1.5	5.47 (0.81)	5.58 (0.59)	6.30 (0.71)
2	5.07 (0.82)	4.82 (0.74)	5.43 (0.59)
2.5	4.24 (0.72)	4.78 (0.75)	5.27 (0.70)
3	4.12 (0.95)	4.34 (0.65)	4.63 (1.24)
3.5	3.52 (0.80)	3.83 (0.99)	4.37 (1.00)
4	2.35 (0.65)	3.03 (0.79)	3.93 (0.34)
4.5	1.29 (0.61)	1.70 (0.46)	2.92 (0.35)
5	1.10 (0.30)	1.40 (0.49)	1.85 (0.36)

Notes: Obtained from 500 simulations of the scale-free network. Standard deviations are reported in parentheses.

Figure 4 The Sensitivity of the Number of Insolvent Banks to Changes in the Ratio of Interbank Debt to Total Senior Liabilities and Interest Rate Based on Simulated Scale-Free Networks



4.1 SI Simulations

Assuming a fixed default probability (DP) for all banks in the system, simulations for both networks investigate how interbank contagion spreads over the network. Simulations thus consider that all banks share an identical balance sheet with homogeneous characteristics. It is also assumed that there is the same ratio of loans with junior priority to those with senior priority and the ratio of interbank loans to the total senior loans. Therefore, the assumptions are like the benchmark model. The interbank linkages formed a network with similar properties reported in Table 5.

The simulations initiate with the insolvency shock of the largest bank, or in other words, removing the largest component of the network. Moreover, there is a set of default probabilities consisting of 5 different values from 0.1 to 0.5. Unlike the balance sheet-based model, the scale-free network shows a higher level of stability against a systemic shock than the dense network. In all scenarios, the contagion process takes less time to cause a systemic failure in the dense network. On average, it takes at least one more period for the scale-free network to encounter a total failure, causing the interbank network to collapse compared to the dense network scenarios. Figure 6 shows the contagion process with different values of DP for both networks.

The results support the idea that more interconnections in a network make it more vulnerable when its components share similar characteristics. There are contradicting results in the literature regarding the vulnerability of homogeneous interbank networks when connectivity increases. Some studies find that a higher degree of interconnectedness positively affects network stability (Allen and Gale 2000; Iori et al. 2008), while some document opposite results (Li and He, 2011). Although the results seem insightful, it must be acknowledged that they are not necessarily realistic since homogeneous interbank networks are rarely formed in the real world. Moreover, the results are largely dependent on the choice of parameters. Besides, in the case of a default in the real network, a bank might recover its debt and convert from an insolvent position to a solvent position, which occurs frequently.

Figure 5 The Sensitivity of the Number of Insolvent Banks to Changes in the Ratio of Interbank Debt to Total Senior Liabilities and Interest Rate Based on Simulated Dense Networks

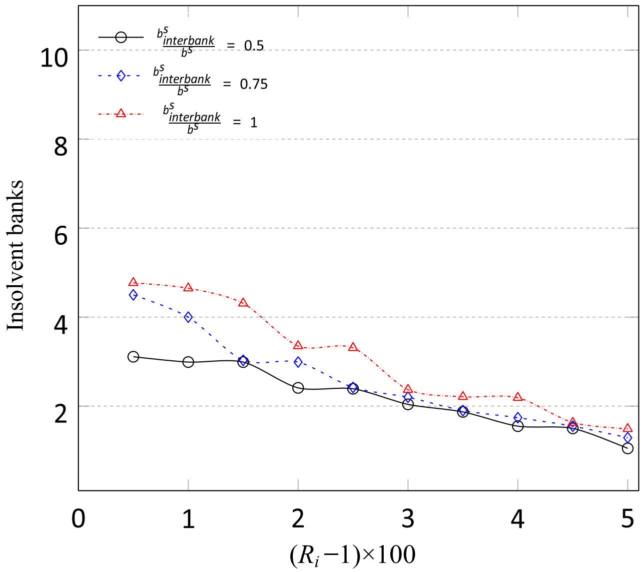
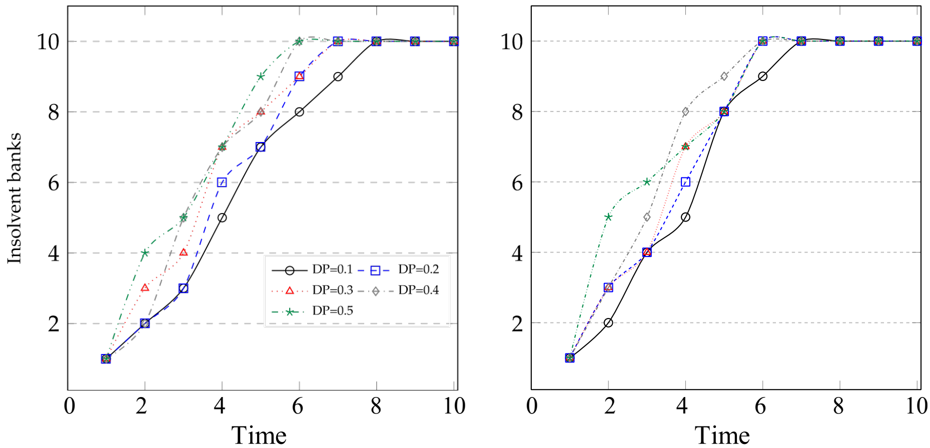


Table 7 Average Number of Insolvent Banks Due to Changes in the Ratio of Interbank Debt to Total Senior Liabilities and Interest Rate

$(R_i - 1) \times 100$	$\frac{b^s_{interbank}}{b^s} = 0.50$	$\frac{b^s_{interbank}}{b^s} = 0.75$	$\frac{b^s_{interbank}}{b^s} = 1$
	# of insolvent banks	*# of insolvent banks*	*# of insolvent banks*
0.5	3.11 (0.94)	4.50 (0.67)	4.77 (1.09)
1	2.99 (0.99)	4.00 (0.45)	4.65 (1.19)
1.5	2.99 (1.03)	3.03 (0.55)	4.31 (0.88)
2	2.41 (0.85)	2.99 (0.55)	3.35 (0.71)
2.5	2.39 (0.58)	2.42 (0.63)	3.33 (0.73)
3	2.04 (0.68)	2.20 (0.75)	2.37 (0.63)
3.5	1.87 (0.69)	1.90 (0.83)	2.21 (0.75)
4	1.55 (0.59)	1.74 (0.82)	2.19 (0.76)
4.5	1.50 (0.50)	1.55 (0.49)	1.63 (0.81)
5	1.05 (0.23)	1.29 (0.46)	1.49 (0.77)

Notes: Obtained from 500 simulations of the dense network. Standard deviations are reported in parentheses.

Figure 6 Simulation Results



Notes: The figure illustrates the results of 500 simulations using the characteristics of the scale-free network (left-hand side) and the dense network (right-hand side).

5. Conclusions

This paper studies contagion processes in a weighted interbank network considering the seniority level of interbank claims. Numerous simulations based on two different network topologies and basic epidemic modeling analyze the contagion process throughout the interbank network in the case of the shock caused by the largest institution's default. This initial shock and its consequences within the interbank network directly address the TCTF paradigm, which a large stream of financial literature investigates. The key finding of this paper is twofold. First, regardless of the network structure, a network consisting of banks with heterogeneous characteristics is more resilient to contagion and systemic defaults when the seniority level of the debt is the same for all institutions in the network. Moreover, a dense interbank network with a higher degree of interconnectedness than the scale-free network exhibits more robustness in the case of contagious defaults. This finding is in line with a significant strand of literature such as Montagna and Lux (2017), documenting the vulnerability of scale-free interbank networks with few money centers having a significant number of connections and a large number of smaller banks with few connections.

Second, the results show that a heterogeneous network with components that have different financial characteristics is resilient to contagion processes when the banks are extensively interconnected. In contrast, a dense network consisting of homogeneous banks with the same financial metrics but with a higher degree of interconnectedness is more vulnerable to systemic shocks and failures than scale-free networks. There are contradicting results in the literature regarding the homogeneity of interbank networks. The results in this paper are consistent with a limited part of the literature documenting that a higher degree of interconnectedness is negatively associated with network resilience (Li and He, 2011). Therefore, these results are worth further investigation.

There are several avenues of research for further studies. With the data on interbank liabilities, empirical research can investigate an optimal network structure and liabilities distribution with the highest resilience to systemic defaults. In addition, future research could also examine other aspects of interbank liabilities, such as the maturity of interbank claims to construct a more realistic interbank network that is empirically tractable.

REFERENCES

Aldasoro I, Alves I (2018): Multiplex Interbank Networks and Systemic Importance: An Application to European Data. *Journal of Financial Stability*, 35:17–37.

Allen F, Babus A, Carletti E (2012): Asset Commonality, Debt Maturity and Systemic Risk. *Journal of Financial Economics*, 104(3):519–534.

Allen F, Gale D (2000): Financial Contagion. *Journal of Political Economy*, 108(1):1–33.

Bargigli L, Di Iasio G, Infante L, Lillo F, Pierobon F (2015): The Multiplex Structure of Interbank Networks. *Quantitative Finance*, 15(4):673–691.

Baruník J, Křehlík T (2018): Measuring the Frequency Dynamics of Financial Connectedness and Systemic Risk. *Journal of Financial Econometrics*, 16(2):271–296.

Birch A, Aste T (2014): Systemic Losses Due to Counterparty Risk in a Stylized Banking System. *Journal of Statistical Physics*, 156(5):998–1024.

Blume L, Easley D, Kleinberg J, Kleinberg R, Tardos É (2011): Network Formation in the Presence of Contagious Risk. In *Proceedings of the 12th ACM conference on Electronic commerce*, pp. 1–10.

Boss M, Elsinger H, Summer M, Thurner S (2004): The Network Topology of the Interbank Market. *Quantitative Finance*, 4:677–684.

Brandi G, Di Clemente R, Cimini G (2018): Epidemics of Liquidity Shortages in Interbank Markets. *Physica A: Statistical Mechanics and its Applications*, 507:255–267.

Bucci A, La Torre D, Liuzzi D, Marsiglio S (2019): Financial Contagion and Economic Development: An Epidemiological Approach. *Journal of Economic Behavior & Organization*, 162:211–228.

Cajueiro D, Tabak B (2008): The Role of Banks in the Brazilian Interbank Market: Does Bank Type Matter? *Physica A: Statistical Mechanics and its Applications*, 387(27):6825–6836.

Chen B, Li L, Peng F, Anwar S (2020): Risk Contagion in the Banking Network: New Evidence from China. *The North American Journal of Economics and Finance*, 54:101276.

Cihak M, Munoz S, Scuzzarella R (2012): The Bright and the Dark Side of Cross-Border Banking Linkages. *Finance a Uver*, 62(3):200.

De Masi G, Iori G, Caldarelli G (2006): Fitness Model for the Italian Interbank Money Market. *Physical Review E*, 74(6):066112.

Degryse H, Nguyen G (2007): Interbank Exposure: An Empirical Examination of Systemic Risk in the Belgian Banking System. *International Journal of Central Banking*, 3(2):123–171.

Dehshalie ME, Kabiri M, Dehshali ME (2021): Stability Analysis and Fixed-Time Control of Credit Risk Contagion. *Mathematics and Computers in Simulation*, 190:131–139.

Diebold FX, Yılmaz K (2014): On the Network Topology of Variance Decompositions: Measuring The Connectedness of Financial Firms. *Journal of Econometrics*, 182(1):119–134.

Eisenberg L, Noe TH (2001): Systemic Risk in Financial Systems. *Management Science*, 47(2):236–249.

Erkol N, Rösch A, Schmidbauer H, Uluceviz E (2016): The Russian Stock Market During the Ukrainian Crisis: A Network Perspective. *Czech Journal of Economics and Finance*, 66(6):478–509.

Fiala T, Havranek T (2017): The Sources of Contagion Risk in a Banking Sector with Foreign Ownership. *Economic Modelling*, 60:108–121.

Fukui M, Furukawa C (2020): Power Laws in Superspreading Events: Evidence from Coronavirus Outbreaks and Implications for sir Models. *MedRxiv*.

Gabrieli S, Salakhova D (2019): Cross-Border Interbank Contagion in the European Banking Sector. *International Economics*, 157:33–54.

Gai P, Haldane A, Kapadia S (2011): Complexity, Concentration and Contagion. *Journal of Monetary Economics*, 58(5):453–470.

Georg C-P (2013): The Effect of the Interbank Network Structure on Contagion and Common Shocks. *Journal of Banking & Finance*, 37(7):2216–2228.

González-Avella JC, de Quadros VH, Iglesias JR (2016): Network Topology and Interbank Credit Risk. *Chaos, Solitons & Fractals*, 88:235–243.

Haldane AG, May RM (2011): Systemic Risk in Banking Ecosystems. *Nature*, 469(7330):351–355.

Hurd TR, Gleeson JP, Melnik S (2017): A Framework for Analyzing Contagion in Assortative Banking Networks. *PLoS One*, 12(2):e0170579.

Iori G, De Masi G, Precup OV, Gabbi G, Caldarelli G (2008): A Network Analysis of the Italian Overnight Money Market. *Journal of Economic Dynamics and Control*, 32(1):259–278.

Kanno M (2015): The Network Structure and Systemic Risk in the Japanese Interbank Market. *Japan and the World Economy*, 36:102–112.

Kermack W, McKendrick A (1927): A Contribution to the Mathematical Theory of Epidemics. In *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, volume 115, pages 700–721. The Royal Society.

Klinger T, Teply P (2016):` The Nexus Between Systemic Risk and Sovereign Crises. *Czech Journal of Economics and Finance*, 66(1):50–69.

Krause A, Giansante S (2012): Interbank Lending and the Spread of Bank Failures: A Network Model of Systemic Risk. *Journal of Economic Behavior & Organization*, 83(3):583–608.

Langfield S, Liu Z, Ota T (2014): Mapping the UK Interbank System. *Journal of Banking & Finance*, 45:288–303.

Lenzu S, Tedeschi G (2012): Systemic Risk on Different Interbank Network Topologies. *Physica A: Statistical Mechanics and its Applications*, 391(18):4331–4341.

Leventides J, Loukaki K, Papavassiliou VG (2019): Simulating Financial Contagion Dynamics in Random Interbank Networks. *Journal of Economic Behavior & Organization*, 158:500–525.

Li S, He J (2011): Resilience of Interbank Market Networks to Shocks. *Discrete Dynamics in Nature and Society*, 2011.

Liu A, Paddrik M, Yang SY, Zhang X (2020): Interbank Contagion: An Agent-Based Model Approach to Endogenously Formed Networks. *Journal of Banking & Finance*, 112:105191.

Martinez-Jaramillo S, Alexandrova-Kabadjova B, Bravo-Benitez B, Solórzano-Margain J (2014): An Empirical Study of the Mexican Banking System's Network and its Implications for Systemic Risk. *Journal of Economic Dynamics and Control*, 40:242–265.

Mitchener KJ, Richardson G (2019): Network Contagion and Interbank Amplification During the Great Depression. *Journal of Political Economy*, 127(2):465–507.

Montagna M, Lux T (2017): Contagion Risk in the Interbank Market: A Probabilistic Approach to Cope with Incomplete Structural Information. *Quantitative Finance*, 17(1):101–120.

Müller J (2006): Interbank Credit Lines as a Channel of Contagion. *Journal of Financial Services Research*, 29(1):37–60.

Nier E, Yang J, Yorulmazer T, Alentorn A (2007): Network Models and Financial Stability. *Journal of Economic Dynamics and Control*, 31(6):2033–2060.

Orhun E (2017): Liquidity Networks in Banking. *Finance a Uver: Czech Journal of Economics & Finance*, 67(2).

Philippas D, Koutelidakis Y, Leontitsis A (2015): Insights into European Interbank Network Contagion. *Managerial Finance*.

Rochet J-C, Tirole J (1996): Interbank Lending and Systemic Risk. *Journal of Money, credit and Banking*, 28(4):733–762.

Silva TC, de Souza SRS, Tabak BM (2016): Network Structure Analysis of the Brazilian Interbank Market. *Emerging Markets Review*, 26:130–152.

Silva W, Kimura H, Sobreiro VA (2017): An Analysis of the Literature on Systemic Financial Risk: A Survey. *Journal of Financial Stability*, 28:91–114.

Smerlak M, Stoll B, Gupta A, Magdanz JS (2015): Mapping Systemic Risk: Critical Degree and Failures Distribution in Financial Networks. *PloS one*, 10(7):e0130948.

Soramäki K, Bech ML, Arnold J, Glass RJ, Beyeler WE (2007): The Topology of Interbank Payment Flows. *Physica A: Statistical Mechanics and its Applications*, 379(1):317–333.

Thurner S, Hanel R, Pichler S et al. (2003): Risk Trading, Network Topology and Banking Regulation. *Quantitative Finance*, 3(4):306–319.

Toivanen M (2013): Contagion in the Interbank Network: An Epidemiological Approach. *Bank of Finland Research Discussion Paper*, 1(19).

Tonzer L (2015): Cross-Border Interbank Networks, Banking Risk and Contagion. *Journal of Financial Stability*, 18:19–32.

Upper C (2011): Simulation Methods to Assess the Danger of Contagion in Interbank Markets. *Journal of Financial Stability*, 7(3):111–125.