Simulation of a Rebound

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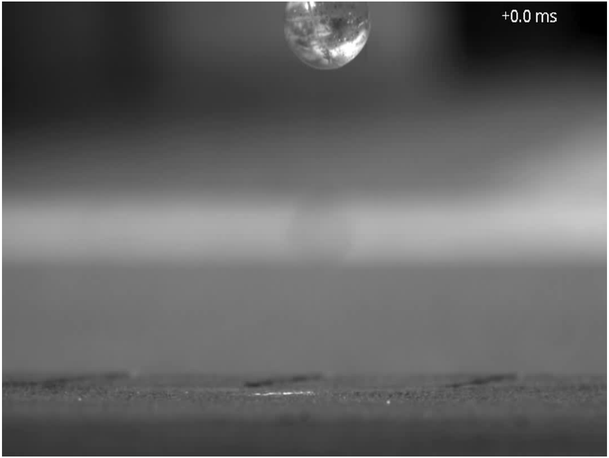
Charles University

Outline

Problem Description



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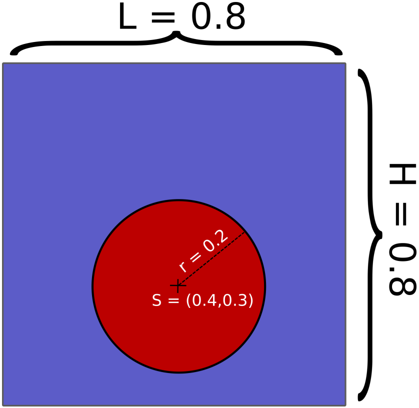
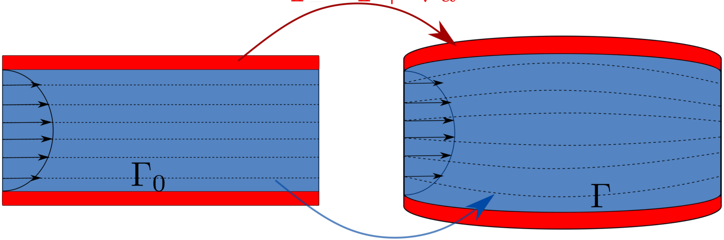
Problem Description

1. 2D
2. elastic material in a viscous Newtonian fluid
3. no additional boundary conditions
4. no gravity field, just initial velocity



ALE method: Description

$u =$ structure deformation
$\mathbb{F} = \mathbb{I} + \nabla u$



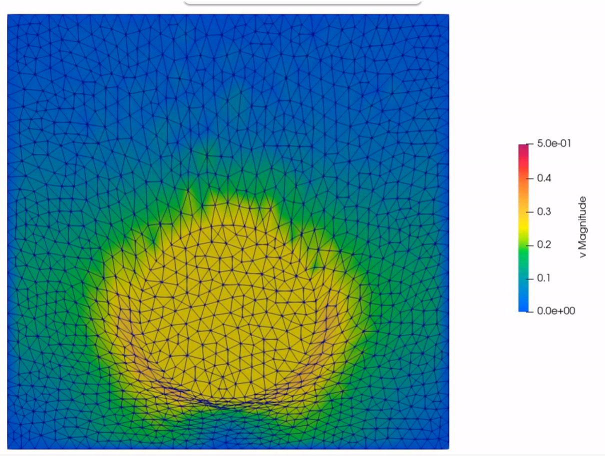
$\mathbb{F}_{\text{ALE}} = \mathbb{I} + \nabla u_{\text{ALE}}$
$u_{\text{ALE}} =$ arbitrary deformation (e.g.$\triangle u_{ALE} = 0$)
$u_{\text{ALE}} = u$ on Γ

ALE method: Advantages

- Sharp boundary
  - We are able to change the interface conditions
  - The mesh does not have to be fine along the interface
- Computation domain is fixed
- The solid is computed in Lagrangian description and fluid in "deformed Eulerian" description

ALE method: Limitation

- Equations are highly nonlinear
- No topological changes
- The mesh projection can be "damaged"
  - $\Rightarrow$ numerical instability



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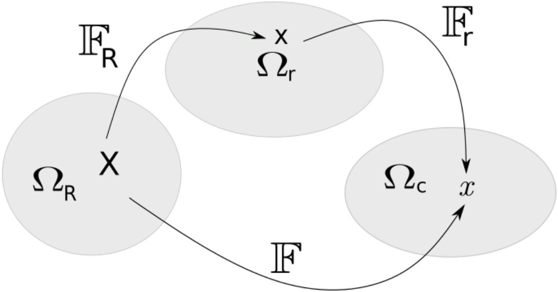
- no-slip on the wall
- no-slip on the interface
- "flat" shape of the structure

$\Rightarrow$ NO CONTACT !!! [1]

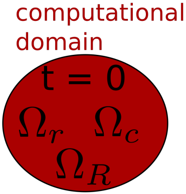
[1] **HESLA T.I. 2004** Collisions of smooth bodies in viscous fluids: a mathematical investigation. PhD thesis, University of Minnesota.

Re-meshing: Re-meshed configuration

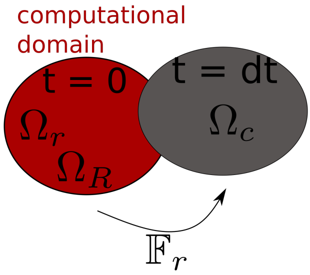


- Ω_R...initial configuration
- Ω_r...re-meshed configuration
- Ω_c...current configuration

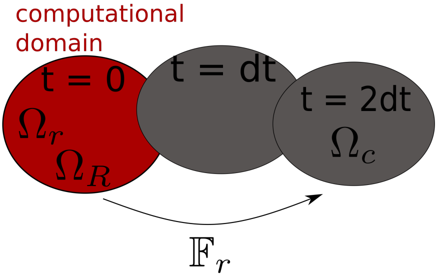
Re-meshing: Re-meshed configuration



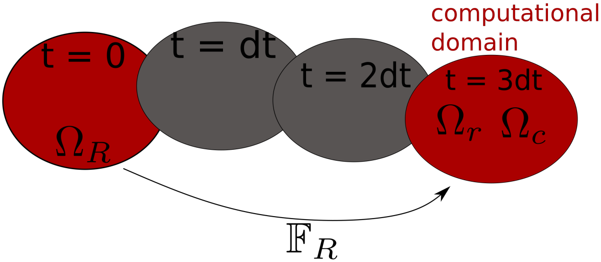
Re-meshing: Re-meshed configuration



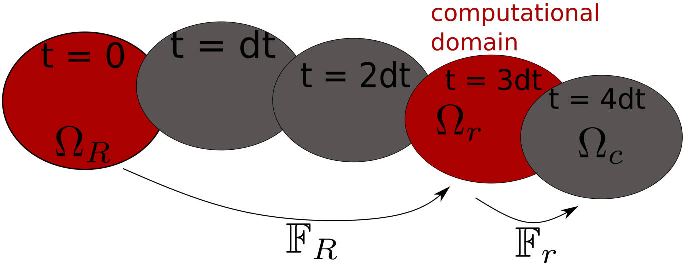
Re-meshing: Re-meshed configuration



Re-meshing: Re-meshed configuration



Re-meshing: Re-meshed configuration



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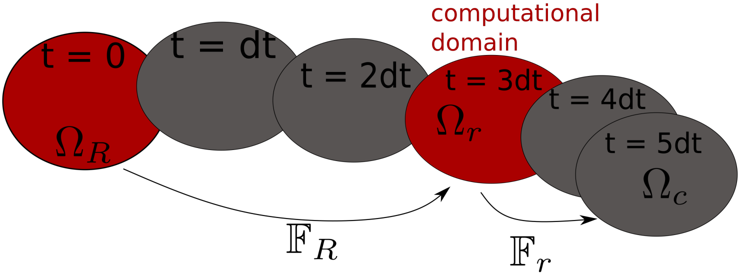
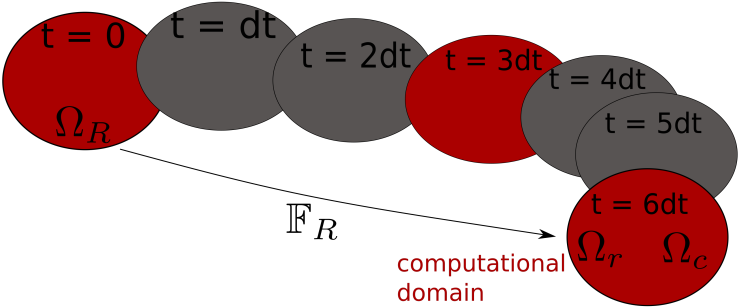
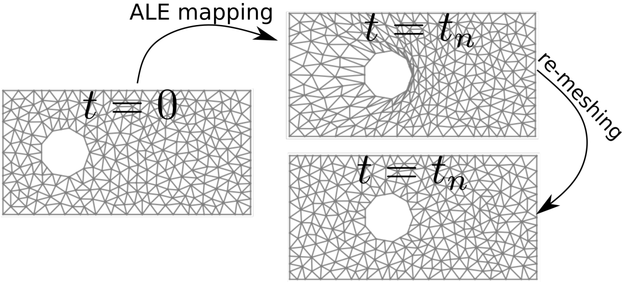


Re-meshing: Re-meshed configuration



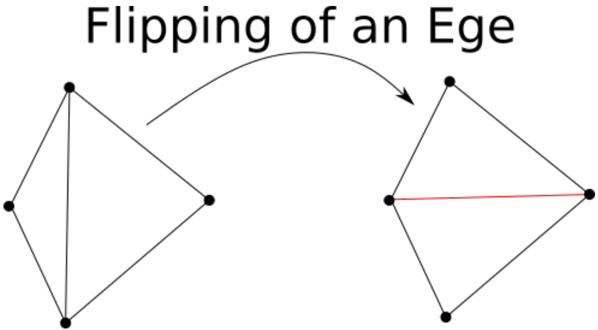
Re-meshing

- Every time we change the computation domain, we repair the mesh and project the solution on it

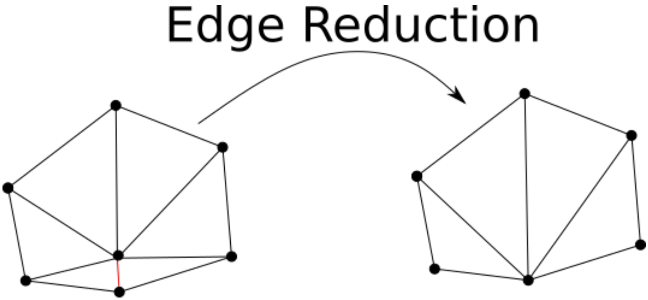


Re-meshing: Local mesh operations

- We will change the mesh locally, where it is needed
- We need to keep the interface
- The mesh can be build just ones
- The number of operations is $\mathcal{O}(n)$, where n is number of elements

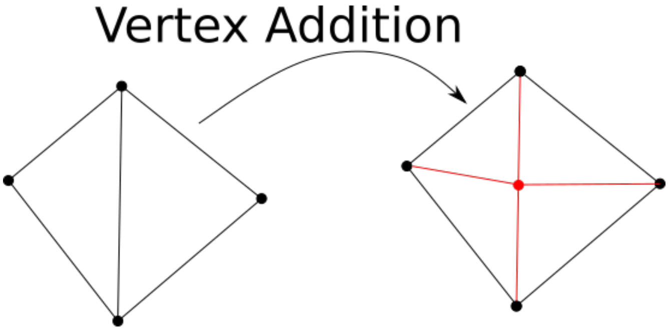

Flipping of an Ege

- Flipping of an Edge



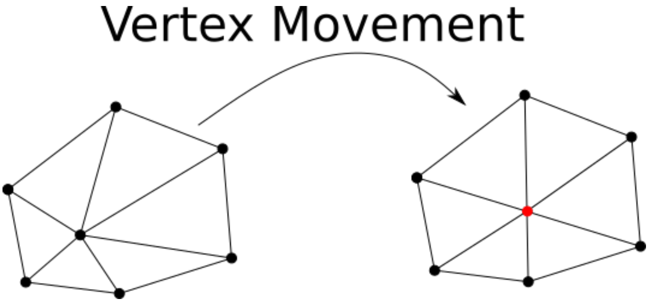
Re-meshing: Local mesh operations

- Flipping of an Edge
- Edge reduction



Re-meshing: Local mesh operations

- Flipping of an Edge
- Edge reduction
- Vertex Addition



Equations

Fluid: Incompressible Navier-Stokes

$$J_f \rho_f \left(\partial_t \vec{v}_f + (\nabla \vec{v}_f)\mathbb{F}_f^{-1}(\vec{v}_f - \partial_t \vec{u}_f)\right) = \operatorname{div}\left(J_f \mathbb{T}_f \mathbb{F}_f^{-T}\right)$$
$$\operatorname{div}(\vec{v}_f) = 0; \quad \triangle \vec{u}_f = 0 \tag{1}$$
$$\mathbb{T}_f = 2\mu\mathbb{D} - p\mathbb{I}$$

Solid: Compressible neo-Hookean

$$\rho_R \partial_t \vec{v}_s = \operatorname{div}\left(J_s \mathbb{T}_s \mathbb{F}_s^{-T}\right)$$
$$\rho_R J_s = \rho_s \quad \partial_t \vec{u}_s = \vec{v}_s \tag{2}$$
$$J_s \mathbb{T}_s \mathbb{F}_s^{-T} = 2G(\mathbb{F}_s - \mathbb{F}_s^{-T}) + 2\lambda(J_s - 1)J_s \mathbb{F}_s^{-T}$$

Equations

Interface conditions

$$\begin{aligned} \vec{u}_f &= \vec{u}_s \text{ on } \Gamma \\ \vec{v}_f &= \vec{v}_s \text{ on } \Gamma \\ \mathbb{T}_f \vec{n} &= \mathbb{T}_s \vec{n} \text{ on } \Gamma \end{aligned} \tag{3}$$

Boundary conditions

$$\begin{aligned} \vec{u} &= 0 \text{ at } \partial\Omega \\ \vec{v} &= 0 \text{ at } \partial\Omega \end{aligned} \tag{4}$$

Numerical Implementation

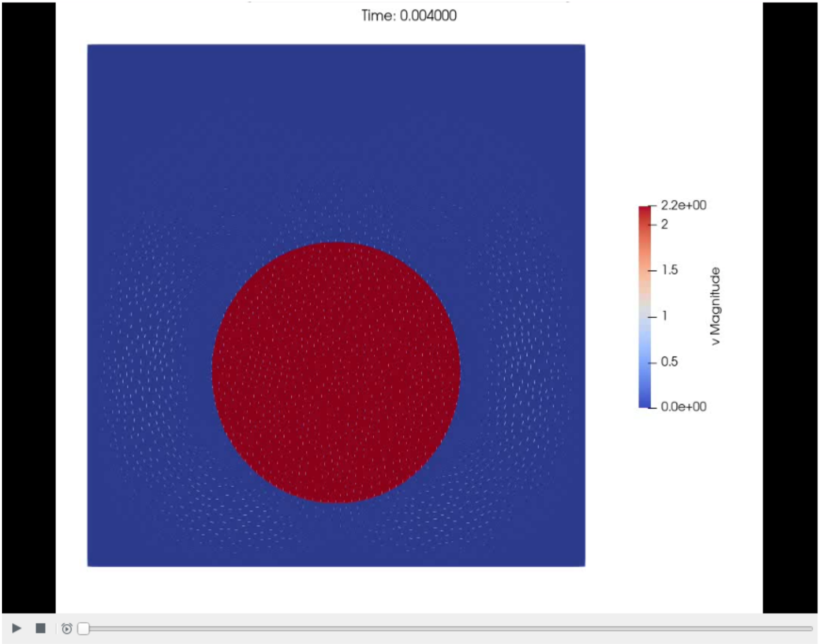
- FEM
- CG2 for $\vec{v}$ and $\vec{u}$
- CG1 for p
- nonlinear Newton solver
- linear solver MUMPS
- rtol $=$ atol $= 10^{-10}$
- $dt = 0.001$, backward Euler

Numerical Implementation

- mesh in ADmesh
- assembled in FEniCS
- solved with petsc4py

Results: Material Parameters

- $\lambda = 10^6 Pa$
- $G = 5 * 10^4 Pa$
- $\mu = 0.2 Pa \cdot s$
- $\rho_f = 1000 \frac{kg}{m^3}$
- $\rho_s = 1000 \frac{kg}{m^3}$



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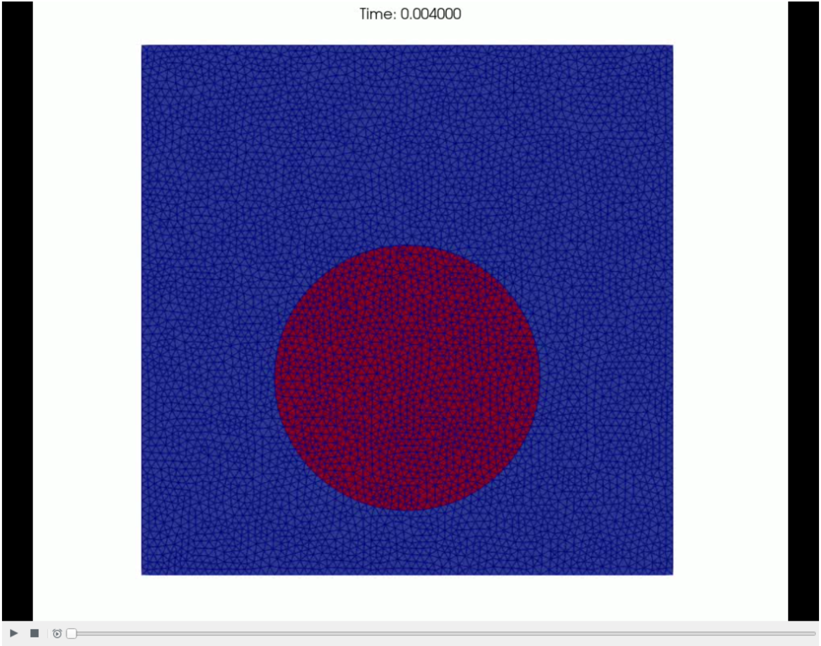

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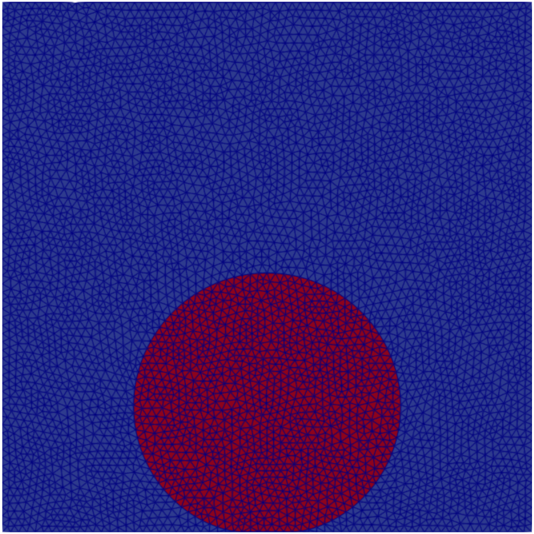
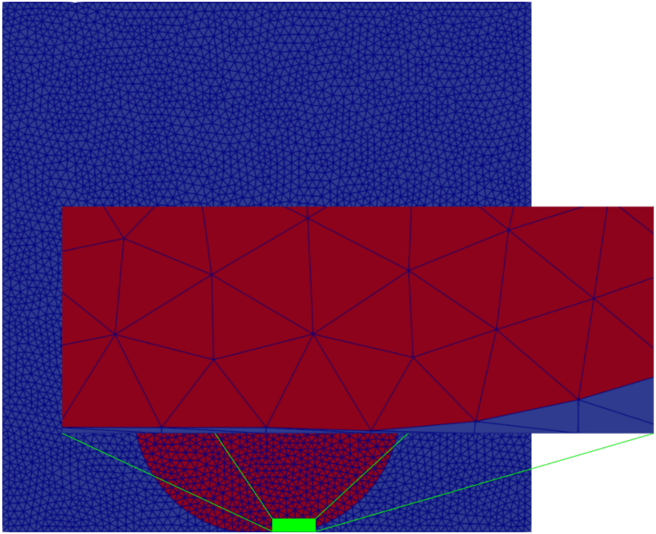


Results: Not Refined Mesh



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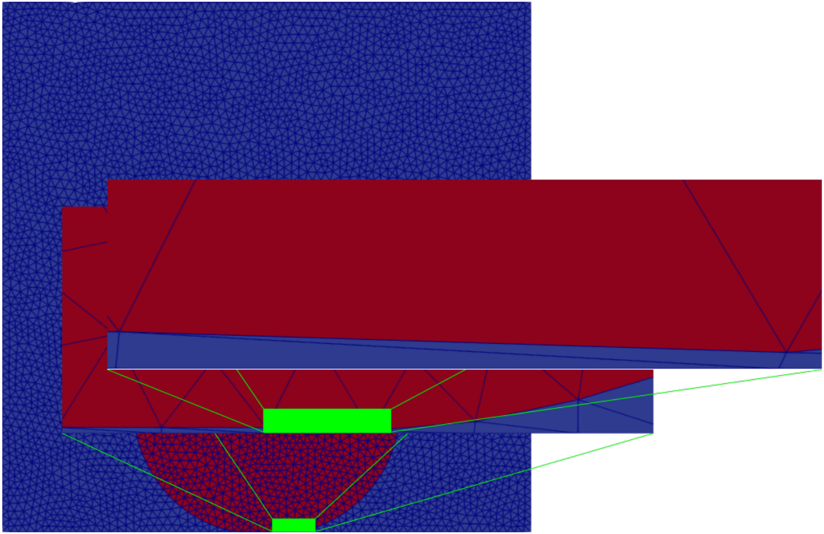
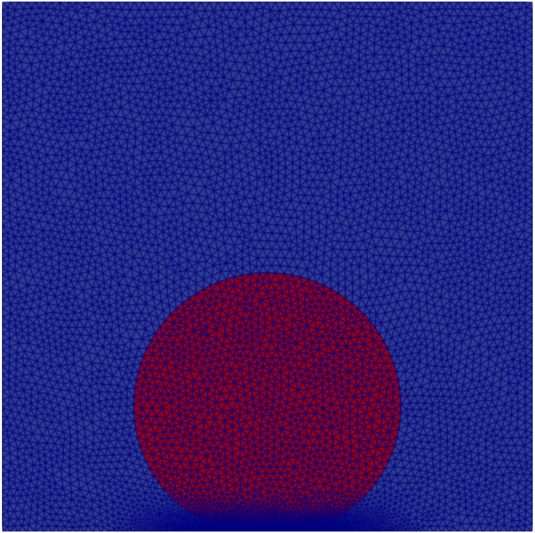


Results: Refined Mesh



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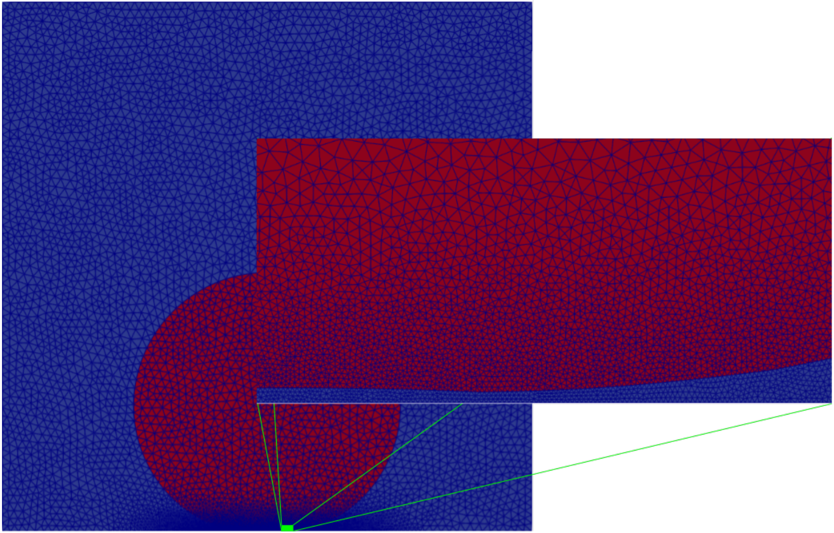
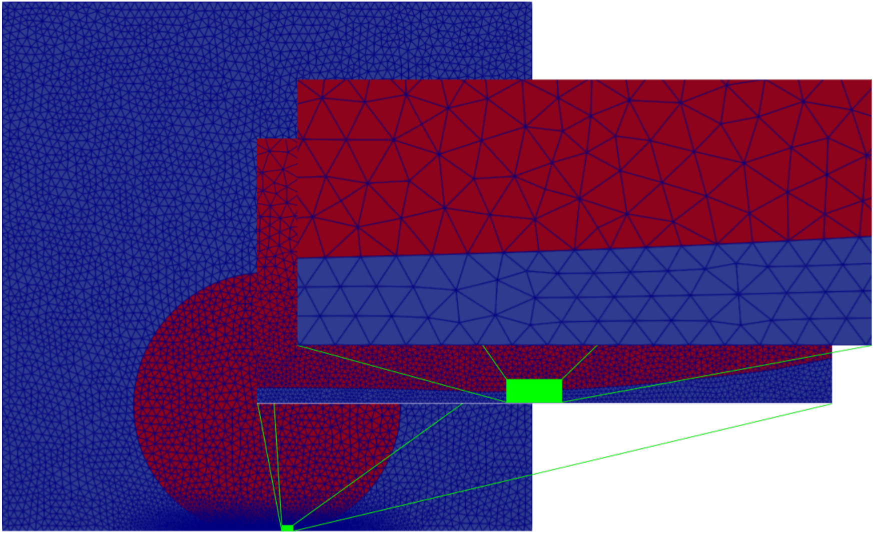
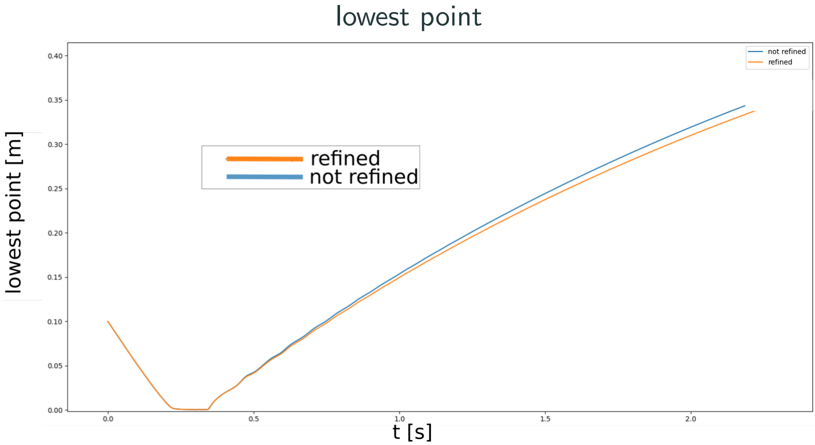
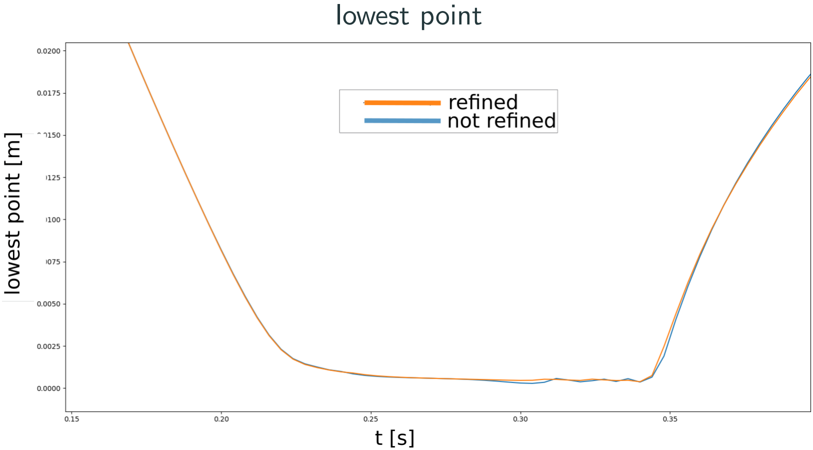


Results: Refined Mesh

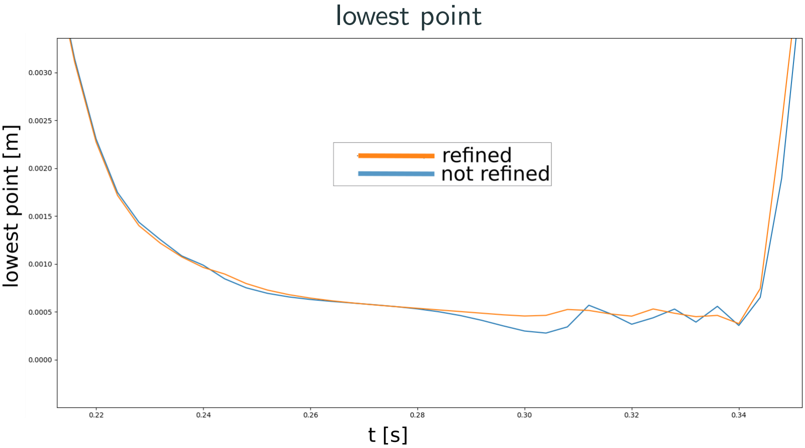


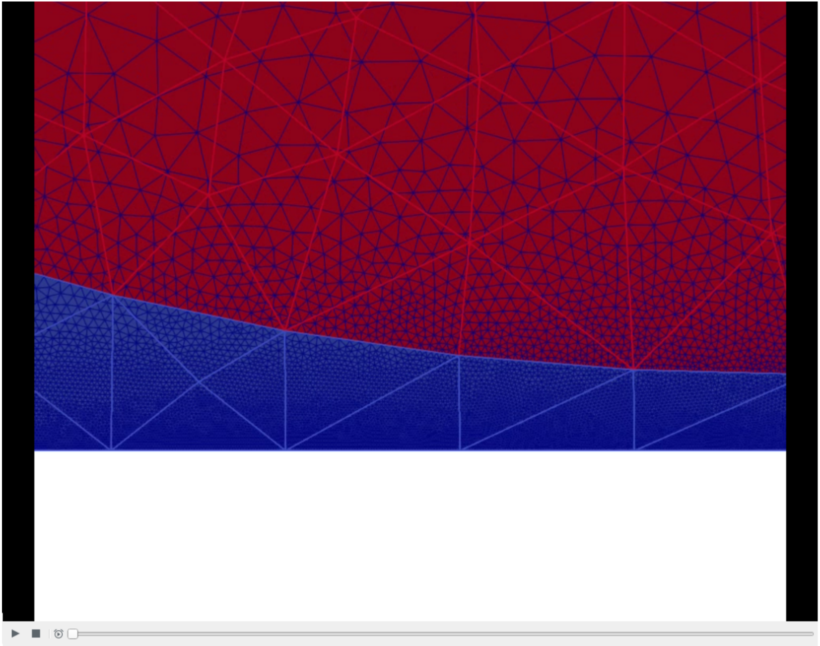

lowest point
lowest point [m]
t [s]
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.0
0.5
1.0
1.5
2.0
not refined
refined
refined
not refined

Results: Comparison



Results: Comparison





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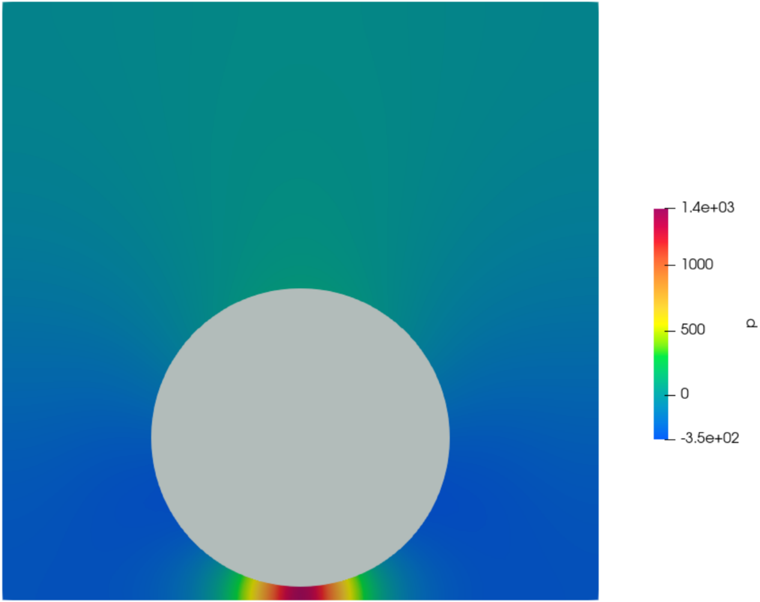
Results: How??

But what stops the ball???

Results: How??



Conclusion

- ALE method can be use for "almost" contact.
- Result does not depend on refinement near the "almost" contact.
- The increment of the pressure causes the rebound.
- We would like to compare results with Eulerian method.